191. Proposed by E. B. ESCOTT, University of Michigan.

Find triangles whose sides are integers and one of whose angles is 60°.

192. Proposed by CHARLES MACAULEY, Chicago, Ill.

Combinations containing an even number of letters are formed of the letters a, b, c, d, etc. It is required to place the letters in two columns, so that half the letters in every combination are placed in one column and the other letters of the combination in the other column, and so that all the a's are placed in the same column; all the b's in the same column; all the c's in the same column, etc.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

383. Proposed by E. B. ESCOTT, University of Michigan.

Find accurately to 6 decimals $1.000000854^{1283451}$.

SOLUTION BY THE PROPOSER.

Using natural logarithms,

$$\log_e(1+x) = x - \frac{x^2}{2} + \cdots.$$

Then

$$\log_e 1.000000854 = .000000854 -, + \cdots,$$

and the natural logarithm of

$$1.000000854^{1283451} \text{ is } 1.096066686.$$

Hence the common logarithm is $1.096066686 \times .434294482$, or .4760157. And the number required is 2.992373.

384. Proposed by H. C. FEEMSTER, York, Nebraska.

A man addressed n envelopes and wrote n checks in payment of n bills. Show that the number of ways of enclosing within each envelope one bill and one check in such a manner that in no instance all the enclosures shall be correct is

$$n!\left\{n! - \frac{(n-1)!}{1!} + \frac{(n-2)!}{2!} \cdots + (-1)^n \frac{0!}{n!}\right\}, \text{ where } 0! = 1.$$

SOLUTION BY THE PROPOSER.

The number of ways in which the enclosures in one letter may be correct is ${}_nC_1((n-1)!)^2$ except ${}_nC_2((n-2)!)^2$ which have been counted twice, except also ${}_nC_3((n-3)!)^2$ of these latter, which have been counted twice, etc. So the number of ways in which the enclosures in one letter may be correct is

$$\begin{aligned}{}_nC_1((n-1)!)^2 - {}_nC_2((n-2)!)^2 + {}_nC_3((n-3)!)^2 - \cdots + (-1)^{n-1}{}_nC_{n-2}(2!)^2 \\ + (-1)^n\, {}_nC_{n-1}(1!)^2 + (-1)^{n+1}\, {}_nC_n(0!)^2.\end{aligned}$$

Hence the number of ways in which both enclosures in a single envelope may be incorrect is:

$$\begin{aligned}(n!)^2 - {}_nC_1((n-1)!)^2 + {}_nC_2((n-2)!)^2 - {}_nC_3((n-3)!)^2 + \cdots + \\ (-1)^{n-2}\, {}_nC_{n-2}(2!)^2 + (-1)^{n-1}{}_nC_{n-1}(1!)^2 + (-1)^n{}_nC_n\, 0!'\end{aligned}$$